Preferred frame and two meanings of time: diagonal form of the *Lorentz-boost* transformation matrix

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Abstract

The main purpose of this paper is to rethink the relativity issue within the framework of the fundamental postulates of quantum mechanics. The aspect of so-called "double special relativity" (DSR) is a starting point in our discussion. The three elementary ideas were involved to show that special relativity may be treated as an integral part of quantum mechanics. These ideas (or observations) are: (1) the necessity of distinguishing the two time meanings, namely: (i) the *vital* one, referring to description of system evolution, and (ii) the *frozen* one, referring to energy measure by means of inverse time units; (2) the existence of the energy-momentum (and time-distance) comparison scale in relativistic description; and (3) a possibility of introduction of mass by means of a light-cone frame description. The resulting quantum-mechanical analysis allows us to find diagonal form of the *Lorentz-boost* transformation matrix and thus to relate the interval invariant relativity principle with the principles of quantum mechanics. The manner, in which the diagonal form of the transformation matrix was found, shows that covariant description itself is a preferred frame description and that the time that undergoes relativistic transformation rules is the *frozen time*, whereas the *vital time* is the Lorentz invariant. A generalized form of the Heisenberg uncertainty principle, proposed by Witten [32], is derived. It turns out, that this form is equivalent to the one know from the analysis of the covariant harmonic oscillator given by Kim and Noz [29]. As a by-product of this analysis one finds that special relativity itself preserves the Planck length, however, a particle cannot be seen any longer as a material point, but rather as an extended quantum object.

1 Introduction

There is no doubt that the special theory of relativity gained the great success in the field of high energy physics, where the local description performed in momentum space dominates. In fact, high energy physics experiment may be "placed" in a middle of energy scale, i.e. much above the energy range proper for the atomic physics and much below the Planck energy, playing the role of fundamental constant in quantum gravity. However, for some time now, it seems that traditional relativity formulation needs a refinement. Some physical examples taken from the "both ends" of the sketched energy scale, seem to confirm such thesis.

In low energy limit we deal with the wide class of, so-called, gedankenexperiments (originated from the well-known Einstein, Podolsky and Rosen paradox [1]), which, by breaking the Bell inequalities [2], manifest a strong nonlocality in quantum mechanics. A very spectacular effect of two-photon entanglement was observed on a distance exceeding 10 km [3]. Another aspect of gedankenexperiment is non-causal information transfer between distant parts of the system [4]. Such behavior is blatantly in contradiction with Einstein locality principle [5], according to which "if S_1 and S_2 are two systems that have interacted in the past but now are arbitrarily distant, the real, factual situation of system S_1 does not depend on what is done with system S_2, which is spatially separated from the former."

An example of another obstacle resulting from orthodox relativity approach concerns a number of problems in cosmology and quantum gravity with fundamental question about the meaning of the Planck length $l_p = \sqrt{hG/\left(2\pi c^3\right)} \sim 1.6 \cdot 10^{-35} m$ (being the combination of the Planck constant h, the gravitational constant G and the speed-of-light c) or its inverse, the Planck energy $E_p = 1/l_p \sim 10^{19} GeV$. For many authors such energy scale acts as a threshold between the known and unknown physical phenomena (e.g.[6]). Therefore, constructing a new theory one expects the traditional relativity approach to break down beyond the threshold but not beneath, where the gravitational field may be weak or even absent. Then, the question asked by Magueijo and Smolin [7] was "in whose reference frame are l_p and E_p the thresholds for new phenomena", because, due to the effect of Lorentz-Fitzgerald contraction the answer is not clear. On the other hand, "it would be paradoxical if different observers disagreed on whether quantum space-time effects are present in a process", as stated by Amelino-Camelia [8]. The class of recently investigated models known as "deformed" or "doubly special relativity" (DSR) [10] is intended to overcome this difficulty.

Actually, the idea of changing the postulates of special relativity by introduction of the second absolute length scale already was introduced by T. G. Pavlopoulos [9], in order to remove divergencies present in relativistic field equations. This idea, next, was rediscovered by Amelino-Camelia [11] where the Planck length scale, another one alongside the speed of light, was introduced, in hope to eliminate a conflict between the assumed fundamental role of the Planck length in the structure of space-time and the Lorentz-Fitzgerald contraction.

Thus, proposed name for the new theory was "double special relativity" [12]. The introduced absolute length scale has set the maximum momentum value (however, with energy has been left unbounded).

In the approach given in [7] the absolute energy and/or length scale was intrinsically built-in into special relativity and set both the maximum energy and momentum. The construction was based on modified action of the Lorentz group acting on momentum space. This action, in general, is nonlinear but reduces to the usual linear action in low energy limit. However, the introduction of the absolute energy scale confronts the complete relativity of inertial frames. On the other hand, the maintenance of complete relativity is one of the major purpose of DSR models [12, 7], what induces the search for general argument for relativity modification other then the existence of a preferred frame. One of the claim is that DSR models should emerge from a fundamental quantum gravity theory [13]. Another possibility was created by so-called varying speed of light (VSL) theories. For example, the VSL model discussed in [14], with a frequency dependent speed of light provided a basis for invariant energy (or length) scale description. The both approaches, however, are burden with difficulties the source of which in large extent is the same, namely, non-linear relativity formulation (also) in position space [15]. To simplify, in "gravity approach" one meets a problem of metric definition, where, in the case of non-liner relativity realization at high energies (or low distances), the concept of metric disintegrates [7]. The VSL model, in turn, involves a problem of consistent velocity determination simultaneously in momentum and in position space. Another complication resulting from the loss of linearity provide descriptions of many-particle systems, for which the kinematic relations of single particles are not valid. So, the difficulties related to fixing the problems generated by non-linear relativity, one may say, exceed somehow the very problem of paradox being the source of DSR approach. The numerous papers cited in [14] gives voice to this.

A formulation based on a preferred frame concept may be an alternative to DSR theory. As suggested in [16] a serious candidate for the preferred frame is the cosmological frame. However, most physicists do not like preferred frames. Why? In opinion of Magueijo [14] "this is more due to mathematical or esthetical reasons then anything else: covariance and background independence have been regarded as highly cherished mathematical assets since the proposal of general gravity". But the other possible answer might be that, so far, the concept of preferred frame is simply misunderstood and, in fact, there is no need to refer to the one particularly chosen physical frame to make use of the *preferred frame description.* Such point of view, of course, gives rise to a basic question about the meaning of the Lorentz symmetry. This paper gives enough simple and transparent arguments, which should convince that usual Lorentz covariant description refers, first of all, to the *preferred frame*, namely the *rest frame of the observer*. Such frame is the only one where all physical measurements can be done. Furthermore, there is even no need to touch on gravity issue since, as it will be shown, the relativity aspect naturally emerges as a consequence of the fundamental postulates of quantum mechanics, and thus it becomes a basic component of particle-wave duality description. If one looks at special relativity

as an integral part of quantum mechanics, one finds that there is no need to interfere within the well-known relativistic formulas to reconcile the absolute energy scale with the Lorentz invariance, as well as, the relativity principle with the existence of the *preferred frame*. Indeed, one finds that special relativity itself preserve the Planck length, however, the particles cannot be seen any longer point-like but rather as extended quantum objects.

The structure of the paper is as follows. In Section 2 there are discussed some general arguments testifying that relativistic description itself is the *preferred frame description*. In Section 3 it is shown how the principles of quantum mechanics combined with the simplest (linear) dispersion relation lead to the interval invariant relativity principle. This analysis is preceded by a discussion which explains the two meanings of time and an idea of *relative scaling*. In Section 4 it is pointed out the importance of a "light-cone skeleton" i.e. the two light cone vectors, which transformed in a unitary way to the Mikowski frame, describe four-momentum of particle with mass. Next, the diagonal form of the Lorentz-boost transformation matrix is derived. The quantum-mechanical foundations of space-time are discussed as well. In Section 5 a velocity problem is considered within the context of Heisenberg uncertainty principle and an arising picture of particle as an extended quantum object. This provide us a new kinematical meaning for the Minkowski space-time. Finally, in Section 6 a meaning of so-called proper time is reexamined.

2 The relativity of inertial frames and the *preferred frame*

The special theory of relativity is based on two postulates: (1) the postulate of relativity of motion and (2) the postulate of the constancy of the speed of light. As already mentioned, there are the attempts to challenge the latter. However, even if there are physical evidences of varying the values of fundamental constants [17] the meaning of such observations, although very instructive, is presumably not strong enough to disrupt essentially traditional overtone of the special relativity. Though the paradox of Magueijo and Smolin concerns mainly the momentum space observation, it corresponds to the well known one of position space, namely, the twin paradox. Therefore, the corrections introduced into the relativity, based on the varying speed of light only, may not be up to the task of refinement of complex relativity issue (especially in the case of "pure relativity analysis", i.e. when additionally no gravity effects are taken into the consideration).

Let us examine then the first relativity principle, which says [18] "the laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of coordinates in uniform translatory motion". This principle firstly expresses our faith in universal character of nature laws. However, our classical perception has led also to the classical conclusion that "there is a transformation group that

converts measurements made by one inertial observer to measurements made by another" [7]. Note, that this "principle-conclusion" cannot be directly verified by any experimental technique, and has become the source of difficulties in relativity interpretation. To approach the matter, let us consider first the case of single observer and ask: is it possible to perform an experiment in which the speed of light, or any other physical quantity, can be measured outside the rest frame of the observer? According to Bell [19] "the only 'observer' which is essential in orthodox practical quantum theory is the inanimate apparatus which amplifies microscopic events to macroscopic consequences". But the 'observer' always observes in 'his' own rest frame and there is no other possibility. So, the rest frame of the observer, identified now with the laboratory frame, plays the role of the *preferred frame,* where all the laws of nature are being discovered and described. In the case of two observers the first relativity postulate could be verified only when the two observers would measure the *same quantity* at the *same time*. Whatever it means, it is clear that now the observers have to be coupled. Thus, they are not independent any longer and the case of two observers reduces simply to the single observer case.

Of course, the same physical phenomena may be observed by independent observers placed in different inertial frames, and we know that their theoretical predictions resulting from the same equations (although established for different boundary conditions) agree. But this is just the case when the *relativity of inertial frames* manifests itself by the fact that *all (inertial) observers that make measurements in the same conditions obtain the same experimental results*. In consequence, the world seen by each of the observers looks the same. In other words, the relativity principle reflects the most basic property of physical observation, namely, its ability to reproduction. On the other hand, the symmetry of derived wave equations (e.g. Maxwell equations) may include a feature of *preferred frame description* itself. Although, in general, the choice of reference frame is free, so that there is no preferred state of motion, form the experimental point of view the velocity is not a purely relative quantity, despite what is commonly thought in a spirit of Einstein interpretation.

3 Two meanings of time and energy-momentum comparison scale

The most natural definition of time characterizes time as a measure of pace of observed changes. We will refer to the time used in such a meaning as to the *vital-time*. The time evolution of system should follow then a *vital-time description.* The quantum mechanics, however, indicates another time meaning. Due to the postulate of Planck, generalized later by Einstein, the time may be also used as the energy measure by means of inverse time units. Since the energy of particle characterized by the wave of period T (or frequency ω) is

$$\mathcal{E} = \frac{h}{T} = \hbar\omega, \tag{1}$$

eq. (1) may also serve as definition of the *frozen-time* being the energy measure. Similarly, due to the postulate of de Broglie, the value of particle momentum Π corresponds to the wavelength λ according to

$$\Pi = \frac{h}{\lambda}. \tag{2}$$

These two fundamental postulates provide the rules for relating energy with time and momentum with length. On the other hand, the momentum and energy, similarly as the distance and time, are the two quantities that cannot be directly compared, because they are expressed in different physical units. Therefore, to compare them in the direct way one needs to introduce a dimensional factor of velocity and thus set a *comparison scale*. One may ask then, whether the time that enters the description which expresses time in length units is *vital* or *frozen?* An example discussed below shows that these two time meanings, in general, are not equivalent.

3.1 The most simple dispersion relation

One of the basic criterion for construction of particle wave equation is the dispersion relation that must be obeyed. The simplest physical scenario provides the situation in which the energy of particle (or quasiparticle) $\mathcal{E}$ is proportional to the momentum value Π. As mentioned, in such a case the dispersion relation must involve some velocity v and may be put down in the form

$$\mathcal{E} = \sigma v \Pi, \tag{3}$$

where $\sigma = \pm 1$, was introduced to allow for the positive and negative energy values. Note, that arbitrarily chosen velocity value imposes the *comparison scale* between the energy and momentum but, in general, it cannot be identified with "true" particle velocity. On the other hand, since the choice of v is assumed to be free, one deals, in fact, with the set of equivalent dispersion equations: $\mathcal{E} = \sigma v_1 \Pi$, $\mathcal{E} = \sigma v_2 \Pi, ...,$ differing just by the velocity factor. This observation may be given in a simpler way by noticing that any velocity value v may be expressed by means of some preferred velocity c and a scaling factor $\eta > 0$ as $v \equiv v_\eta = \eta^2 c$. It allows to introduce the formula

$$\frac{1}{\eta}\left(\frac{\mathcal{E}}{c}\right) = \sigma \eta \Pi, \tag{4}$$

in which given value η corresponds to the one of dispersion equation taken from the set.

Next, let us assume that it is possible to find a physical system characterized by the linear energy-momentum dependence and the quasiparticle excitations, which always propagate with a constat velocity c in a given medium (no matter how big such excitations are). In this case, the most natural dispersion relation is given by eq. (4) for $\eta = 1$. In effect, such an idealized system may serve as

the reference one to set the *mappings* for energy

$$\mathcal{E} \rightarrow \frac{\mathcal{E}}{c} = \Pi_0, \tag{5}$$

and time

$$t \rightarrow x_0 = ct. \tag{6}$$

Obviously, such idealized system exists and is well-known, so it will be simply called the *photon system.* Note, that eqs. (1), (2) and (3) for $\sigma = 1$ and $\eta = 1$ yield the classical dispersion relation $\lambda = cT$, valid for single electromagnetic monochromatic plane wave. Thus, naively speaking, in the time period T a photon makes the distance

$$\Delta x_1 = \lambda = cT = \Delta x_0. \tag{7}$$

Relations (7) show that in the case of the *photon system* the time interval $\Delta x_0/c$ has clear kinematical and dynamical meaning, so that the *vital* and *frozen* time meanings can be identified. However, when particle velocity depends on energy (or momentum) the relationship between the *vital* and *frozen* time intervals is unknown, and the equivalence between the two time meanings no longer holds. Of course, one may argue that if particle velocity is known, then for a given *frozen* time interval the corresponding *vital* one is known too. But the key point is that in the problems based on momentum-space (position-space) description the velocity (energy) is a quantity, which in principle is unknown. Roughly speaking, quantum effects hidden under the cover of the Heisenberg uncertainty principle are the reason for that. Note, that in order to establish in experimental way the relationship between particle velocity and energy, one has to make the simultaneous and independent measurement of both quantities. In particular, the velocity has to be established in time-of-flight (TOF) method. Such measurement, however, must violate somehow the Heisenberg uncertainty principle

$$\Delta E \Delta t \gtrsim h, \tag{8}$$

where Δt and ΔE are the uncertainties of time and energy related to the measurement. We will return to this issue latter, when a generalized form of the Heisenberg principle will be discussed. Note also that Heisenberg uncertainty principle does not concern the *photon system* because of the postulate of the constancy of the speed of light, so the *photon system* seems to be favored once again.

Elementary analysis given above comes to the conclusion that the notion of space-time refers to the space, which is reciprocal to energy-momentum one, or in other words, that both spaces of momentum and position play the role of *reciprocal spaces* in traditional relativity formulation. Therefore, the time that enters the space-time description, in general, may be identified only with the *frozen time.*

3.2 The energy-momentum and time-distance *relative scaling*

We now consider the meaning of the scaling factor η introduced in (4). For $\eta = 1$ one obtains the *photon* dispersion relation

$$\Pi_0 = \sigma\Pi. \tag{9}$$

In the case of $\eta \neq 1$, eq. (4) can be still written in the same form

$$\Pi_0' = \sigma\Pi', \tag{10}$$

where

$$\Pi_0' = \frac{1}{\eta}\Pi_0 \quad \text{and} \quad \Pi' = \eta\Pi, \tag{11}$$

are the values of energy and momentum considered, however, in the frame which axes have been *relatively re-scaled* with respect to the reference frame given for $\eta = 1$. Similarly, according to (1), (2) and (7) the scaling conditions (11) written in space-time notation are:

$$\Delta x_0' = \eta\Delta x_0 \quad \text{and} \quad \Delta x_1' = \frac{1}{\eta}\Delta x_1. \tag{12}$$

So, for $\eta > 1$, the transformations (12) may be called "time dilatation" and "length contraction". We note also that dispersion eqs. (9) and (10) are in relation

$$\Pi_0^2 - \mathbf{\Pi}^2 = \Pi_0'^2 - \mathbf{\Pi}'^2 = 0, \tag{13}$$

which written in space-time notation takes the familiar form

$$(\Delta x_0)^2 - (\Delta x_1)^2 = (\Delta x_0')^2 - (\Delta x_1')^2 \;\; = 0. \tag{14}$$

It is obvious that the formalism based on arbitrarily chosen energy-momentum (or time-distance) *comparison scale* has to be covariant relative to the change of this scale. The scaling factor η that sets the *comparison scale,* plays a role of a "master-parameter" in homogenous Lorentz group. Namely, it will be shown that it "splits" into three boost parameters.

4 The *Lorentz-boost* transformation in diagonal form

The homogenous Lorentz group has six parameters, where three of them refer to the subgroup of three-dimensional rotations. Currently we neglect the very Euclidean aspect of the Lorentz transformations and concentrate only on their most crucial *boost-part*, which is going to be reduced to diagonal form. We start from one-dimensional analysis by introducing a concept of *bimomentum,* helpful in description of particle mass. The generalization into three dimensions will be straightforward. We start, however, from pointing out some basic and important features hidden in light-cone frame description.

4.1 *Bimomentum* and introduction of mass

The *photon system,* discussed already, was given as an example of an idealized system in description of which the *vital* and *frozen* time meanings can be identified. In general, such an equivalence of both time meanings applies to relativistic descriptions of zero-mass fields, i.e.: the free electromagnetic field, free scalar field and Weyl fields for spin-half particle. All these fields are characterized by the same light-cone dispersion relation

$$\Pi_0 = \pm |\mathbf{\Pi}| \,, \tag{15}$$

so that, their quasiparticle excitations are assumed to propagate always with the same velocity c. Since the algebraic structure of these (massless) fields now is of no importance, we will call them all the *photon fields.* As already noticed, in the case of material particle, when velocity is energy-dependent, the *vital* meaning of time in space-time description, in general, brakes down. Thus, the understanding of correlation between *vital* and *frozen* time intervals seems to be the key issue in encompassing of relativity matter.

The light-cone frame description, of course, goes much beyond the formulation of dispersion relation for massless particles. It is well-known the usefulness of light-cone frame description in quantization of strings in the string theory [25]. A characteristic feature of this light-cone frame description is, that it appears in problems where particle mass comes not as a feature of wave equation but rather its solutions. Indeed, another example provides so-called covariant harmonic oscillator, discussed widely by Kim and Noz beginning with [26]. The issue of the covariant harmonic oscillator originally was considered in a context of description of relativistic hadrons, but, in general, one may say that it touches a problem of description of composite quantum structures. Indeed, it was shown that the wave equation given by Feynman, Kislinger and Ravndal [27], proposed to describe a self-interacting hadron, separates into the Klein-Gordon equation and the covariant harmonic oscillator one [28]. As a result, the Klein-Gordon particles have masses which correspond different solutions of the covariant harmonic oscillator equation.

In other words, the plane-wave properties of Klein-Gordon particles have their origin in a quantum structure of bounded (particle) states. An arising picture of particle-wave duality coming from above is rather clear. Furthermore, although the dispersion relation obeyed by the Klein-Gordon plane-waves is not the light-cone one, it will be shown that it has a unitary light-cone "equivalent". On the other hand, also it will be shown, that the "size" of this oscillatory-like particle may be placed at the same light-cone "structure". In other words, one may indicate some *light-cone skeleton* that unifies a description of particle-wave duality.

Since the main purpose of this paper is only an elementary analysis of the relativity issue within the framework of the postulates of quantum mechanics, some quantum-mechanical aspects related to the solutions of covariant harmonic oscillator [29], will serve us as an illustration to the presented ideas, whereas the field theory analysis concerning this issues will be given separately [20].

To approach the matter, we first consider a concept of *bimomentum* which makes possible to represent the massive and massless particle states by means of the latter ones only.

The *bimomentum* $\binom{\Pi_0}{\Pi_1}$ is defined as the two-component light-cone momentum vector, which components may be referred to two different solutions of *photon* (i.e. massless) equation in the following way: the first component Π_0 is the energy of the first *photon*, whereas the second component Π_1 is the momentum of the second *photon*. The *bivector* represents then a *two-photon state*. Thus, the two *light-cone photons* are assumed to propagate along the same real space axis. For the purpose of this paper it is enough to limit the analysis to the case where both *photons* have positive energies.

Let us next consider a unitary transformation of *bimomentum* into *effective bimomentum* defined as

$$\left(\begin{array}{c} p_0 \\ p_1 \end{array} \right) = \left(\begin{array}{cc} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{array} \right) \left(\begin{array}{c} \Pi_0 \\ \Pi_1 \end{array} \right), \tag{16}$$

where p_0 and p_1 are assumed to be the energy and momentum of physically observed state, i.e. in a frame other then the light-cone one. For particular choice of $\alpha = 45^0$ we will call this frame the Minkowski frame. Thus, we limit our discussion to the case

$$\left(\begin{array}{c} p_0 \\ p_1 \end{array} \right) = \left(\begin{array}{cc} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array} \right) \left(\begin{array}{c} \Pi_0 \\ \Pi_1 \end{array} \right). \tag{17}$$

This transformation corresponds to the one introduced by Dirac [21] and has been already used to demonstrate the changes in four-momentum distribution of boosted covariant harmonic oscillator ground state [30].

In three dimensional real space the *effective bimomentum* $\binom{p_0}{p_1}$ must correspond to the gauge Minkowski four-momentum, which two components equal zero. As an example of transformation (17) it is advisable to consider the two generic cases: (i) $\Pi_1 = 0$, and (ii) $\Pi_1 = -\Pi_0$.

In the first case the *bimomentum* $\binom{\Pi_0}{\Pi}$ represents a single (Minkowski) *photon*, since due to (17)

$$\left(\begin{array}{c} \Pi_0 \\ \Pi_1 \end{array} \right) \underset{\Pi_1=0}{\longrightarrow} \left(\begin{array}{c} \Pi_0/\sqrt{2} \\ \Pi_0/\sqrt{2} \end{array} \right). \tag{18}$$

In the second case, let us put

$$\Pi_1 = \frac{mc}{\sqrt{2}} = -\Pi_0,$$

which gives

$$\left(\begin{array}{c} \Pi_0 \\ \Pi_1 \end{array} \right) \underset{\Pi_0=-\Pi=\frac{mc}{\sqrt{2}}}{\longrightarrow} \left(\begin{array}{c} mc \\ 0 \end{array} \right). \tag{19}$$

Thus, now the *effective bimomentum* corresponds to the state of zero momentum and non-vanishing energy $E_0 = c\Pi_0 \equiv mc^2$, i.e. the ground state energy. One

finds then, that the concept of *bimomentum* allows us to combine in a unitary way the two massless *photon states* into one *effective state,* which may be either massless or massive.

4.2 The *relative scaling* and Lorentz symmetry

It was argued that the scaling parameter η (4) was to be considered as the parameter of *preferred frame description.* It was shown also that the change of η, what corresponds to the change of energy-momentum (11) or time-distance (12) *comparison scale,* preserves the form of the *photon* dispersion relation (9). We now consider the transformation of *effective bimomentum* induced by the *relative scaling* of *bimomentum* components.

Let us assume that eq. (17) is written in the frame given for $\eta = 1$. Thus, in the frame for which $\eta \neq 1$ the form of *effective bimomentum* is given by

$$\begin{pmatrix} p'_0 \\ p'_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \Pi'_0 \\ \Pi'_1 \end{pmatrix}, \tag{20}$$

where, due to (11), the coordinates of new (primed) and old (unprimed) *bimomenta* fulfil

$$\begin{pmatrix} \Pi'_0 \\ \Pi'_1 \end{pmatrix} = \begin{pmatrix} \eta & 0 \\ 0 & 1/\eta \end{pmatrix} \begin{pmatrix} \Pi_0 \\ \Pi_1 \end{pmatrix}. \tag{21}$$

By combining equations (17), (20) and (21) one finds that relationship between the coordinates of *bimomenta* in different Minkowski frames is given by the Lorentz transformation formulas

$$\begin{pmatrix} p'_0 \\ p'_1 \end{pmatrix} = \begin{pmatrix} cosh\xi & sinh\xi \\ sinh\xi & cosh\xi \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \end{pmatrix} \quad \text{where } \xi = ln\eta, \tag{22}$$

This points out to the dependence on the Lorentz symmetry with the freedom of scaling η.

More precisely, the transformation (22) may be understood as: (1) the *passive* one, when both *bi-momenta* $\binom{p_0}{p_1}$ and $\binom{p'_0}{p'_1}$ refer to the one physical state but are described in two different frames (i.e. in the frames based on different *comparison scales*), or (2) as the *active* one when both *bi-momenta* are considered in the same *preferred frame* but refer to two different *physical states.* We will say that in the case (1) η is the *passive* parameter, whereas in the case (2) η is the *active* one. So, in the latter case the $\eta - parametrization$ does not concern the frame characteristics but a dynamical feature of the state. We write this down in the explicit form

$$\begin{pmatrix} p_0 \\ p_1 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} mc \\ 0 \end{pmatrix}, \tag{23}$$

where $\gamma = cosh\xi$ and $\gamma \cdot \beta = sinh\xi$. Formally, the transformation matrixes given in (22) and (23) are identical. However, the parameters γ and β (which

kinematical meaning will be discussed latter) were introduced to emphasis that their values need to be considered with reference to the ground state energy.

Since the ground state energy is characterized by zero momentum, the transformation (23), generalized into the three-dimensional case, should take the form

$$\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} mc \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{24}$$

which describes a boost in the x direction ($p_2 = p_3 = 0$). The matrix form of eq. (24) is

$$P_x = A_0\ mc. \tag{25}$$

To describe a boost in any direction one needs to rotate the initial frame, first (let say) in xy and next in yz plane. In the new frame eq. (25) takes the form

$$R_{yz}R_{xy}P_x = R_{yz}R_{xy}A_0R_{xy}^{-1}R_{yz}^{-1}R_{yz}R_{xy}mc, \tag{26}$$

where

$$R_{xy} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi & 0 \\ 0 & sin\varphi & cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},\ R_{yz} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & cos\psi & -sin\psi \\ 0 & 0 & sin\psi & cos\psi \end{pmatrix}, \tag{27}$$

so that, the angles φ and ψ describe the rotations around the z and x axes respectively. The explicit form of eq. (26) is then given by

$$\begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta_1 & \gamma\beta_2 & \gamma\beta_3 \\ \gamma\beta_1 & 1+\frac{(\gamma-1)}{\beta^2}\beta_1^2 & \frac{(\gamma-1)}{\beta^2}\beta_1\beta_2 & \frac{(\gamma-1)}{\beta^2}\beta_1\beta_3 \\ \gamma\beta_2 & \frac{(\gamma-1)}{\beta^2}\beta_1\beta_2 & 1+\frac{(\gamma-1)}{\beta^2}\beta_2^2 & \frac{(\gamma-1)}{\beta^2}\beta_2\beta_3 \\ \gamma\beta_3 & \frac{(\gamma-1)}{\beta^2}\beta_1\beta_3 & \frac{(\gamma-1)}{\beta^2}\beta_2\beta_3 & 1+\frac{(\gamma-1)}{\beta^2}\beta_3^2 \end{pmatrix} \begin{pmatrix} mc \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{28}$$

where $\beta_1/\beta = cos\varphi$, $\beta_2/\beta = sin\varphi{\cdot}cos\psi$ and $\beta_3/\beta = sin\varphi{\cdot}sin\psi$. This leads to the formulas for energy $E = \gamma mc^2$ and momentum $\mathbf{p} = \gamma\boldsymbol{\beta}$, where $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)$. Eq. (28) may be written as

$$P = \mathbf{A}\ m. \tag{29}$$

To recollect, the matrix $\mathbf{A}$ represents the *Lorentz-boost* transformation. Traditionally it is derived by means of the boost generators of the Lorentz group and the Taylor expansion [22]. The way it was constructed now allows us to express the matrix $\mathbf{A}$ in the form

$$\mathbf{A} = \mathbf{R}\mathbf{U}\boldsymbol{\Lambda}_\eta\mathbf{U}^{-1}\mathbf{R}^{-1}, \tag{30}$$

where

$$\mathbf{\Lambda}_\eta = \begin{pmatrix} \eta & 0 & 0 & 0 \\ 0 & 1/\eta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; \mathbf{U} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; \mathbf{R} = R_{yz} \circ R_{xy}. \tag{31}$$

Since neither of the matrixes $\mathbf{R}$ nor $\mathbf{U}$ is singular the diagonal form of the matrix $\mathbf{A}$ is given by $\mathbf{\Lambda}_\eta$. So, it was shown that, although there are three different boosts parameters, their origin is provided by the one scaling factor η, established on a ground of purely quantum-mechanical considerations.

4.3 Quantum-mechanical foundations of the Minkowski space

The notion of space-time origins from the classical analysis of the electromagnetic field. Currently we show that the space-time naturally emerges as the *reciprocal* to energy-momentum one.

To recognize the "reciprocal dependence" of position and momentum spaces we start again from the momentum space by considering the scaling transformation (21) acting on *bimomentum* of the ground state, namely

$$\begin{pmatrix} \Pi_0 \\ \Pi_1 \end{pmatrix} = \begin{pmatrix} \eta & 0 \\ 0 & 1/\eta \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} mc \\ -\frac{1}{\sqrt{2}} mc \end{pmatrix}, \tag{32}$$

where η plays the role of *active* parameter. Thus, according to (17), the components of *bi-momentum* related to *bimomentum* $\binom{\Pi_0}{\Pi_1}$ in (32) may be expressed via *active* η as

$$p_0 = \frac{1}{2}\left(\eta + \frac{1}{\eta}\right) mc, \quad p_1 = \frac{1}{2}\left(\eta - \frac{1}{\eta}\right) mc. \tag{33}$$

Note, that the coordinates of both *bimomenta* (32) (and thus the ground state energy mc^2) refer to the *preferred frame description,* for which the *passive* η is assumed to be equal one. One easily finds that Π_0 and Π_1 given in (32) satisfy the condition

$$(\Pi_0 - \Pi_1)^2 - (\Pi_0 + \Pi_1)^2 = (mc)^2, \tag{34}$$

which is invariant with respect to the choice of η. As already mentioned, the mappings (5) and (6) allow us to write down the components of *bimomenta* in terms of the wavelengths, i.e. *generalized Compton wavelength* λ_1 and de Broglie wavelength λ_2 defined as

$$\Pi_0 = \frac{h}{\lambda_1} \quad \text{and} \quad \Pi_2 = -\frac{h}{\lambda_2}. \tag{35}$$

Relations (35) provide then a basis for a transition from the momentum to position space. Thus, in position space one finds

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1/\eta & 0 \\ 0 & \eta \end{pmatrix} \begin{pmatrix} \sqrt{2}\lambda_0 \\ \sqrt{2}\lambda_0 \end{pmatrix}, \tag{36}$$

where due to (32)

$$\lambda_0 = \frac{h}{mc}, \tag{37}$$

is the "proper" Compton wavelength. Eq. (34) written in its reciprocal version, takes the form

$$\left(\frac{\lambda_1+\lambda_2}{2}\right)^2 - \left(\frac{\lambda_1-\lambda_2}{2}\right)^2 = 2\left(\frac{h}{mc}\right)^2 = const, \tag{38}$$

which shows that the right hand side of eq. (38) is (again) an $\eta-$scaling invariant.

Eqs. (32) and (36) are the light-cone equations. Similarly, like in the case of momentum space analysis, one may introduce the Minkowski frame representation for the wavelengths λ_1 and λ_2, namely

$$\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}. \tag{39}$$

Thus, for any two pairs of variables $\{x_0, x_1\}$ and $\{x'_0, x'_1\}$ (corresponding to the two different values of η and η') it must occur

$$x_0^2 - x_1^2 = x_0'^2 - x_1'^2 = 2\left(\frac{h}{mc}\right)^2 = const. \tag{40}$$

The connection between the quantum mechanics and special relativity is obvious. Eqs. (40) states the equivalent of eqs. (14) in the case of non-zero mass. The different definitions of time and space intervals (7) and (39) (distinguishing the massless and massive cases) cause that expression (40) does not reduce to (14) in the limit $m \to 0$. Note, that the quantum aspect of the approach appears explicitly just along with the introduction of mass.

The generalization for three dimensions is similar to the corresponding procedure in the momentum space, but now we are mainly interested in transformation (22) as that *passive* one. Let us consider a particle state of energy p_0 and momentum $\mathbf{p} = (p_1, 0, 0)$. The quantities p_0 and p_1 can be expressed by means of *active* η (33). On the other hand p_0 and p_1, through (17) and (35) can also be expressed in terms of reciprocal quantities x_0 and x_1 given in (39). One may ask then about the space-time transformation, which corresponds to the *active* transition $\binom{p_0}{p_1} \underset{\eta_1\to\eta_2}{\longrightarrow} \binom{p'_0}{p'_1}$. One easily finds that this *passive* transformation $\{x_0,x_1,x_2,x_3\} \to \{x'_0,x'_1,x'_2,x'_3\}$ takes the form

$$\begin{pmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{pmatrix} cosh\xi & -sinh\xi & 0 & 0 \\ -sinh\xi & cosh\xi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}, \tag{41}$$

where $\xi = ln(\eta_1/\eta_2)$. In more general case, when the real-space axes of both frames are parallel but direction of the boost direction $\widehat{\boldsymbol{\beta}}$ does not match any of

the axes directions, the transformed form of eq. (41) is given by

$$\begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta_1 & -\gamma\beta_2 & -\gamma\beta_3 \\ -\gamma\beta_1 & 1+\frac{(\gamma-1)}{\beta^2}\beta_1^2 & \frac{(\gamma-1)}{\beta^2}\beta_1\beta_2 & \frac{(\gamma-1)}{\beta^2}\beta_1\beta_3 \\ -\gamma\beta_2 & \frac{(\gamma-1)}{\beta^2}\beta_1\beta_2 & 1+\frac{(\gamma-1)}{\beta^2}\beta_2^2 & \frac{(\gamma-1)}{\beta^2}\beta_2\beta_3 \\ -\gamma\beta_3 & \frac{(\gamma-1)}{\beta^2}\beta_1\beta_3 & \frac{(\gamma-1)}{\beta^2}\beta_2\beta_3 & 1+\frac{(\gamma-1)}{\beta^2}\beta_3^2 \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{pmatrix}, \tag{42}$$

where

$$\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{pmatrix} = R_{yz}R_{xy} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}. \tag{43}$$

This ensures, of course, that the condition of interval invariance

$$(x_0)^2 - (x_1)^2 - (x_2)^2 - (x_3)^2 = (x_0')^2 - (x_1')^2 - (x_2')^2 - (x_3')^2 = const, \tag{44}$$

is fulfilled. Note, that this condition being the principle of relativity formulation, due to (14) and (40), now, finds a quantum-mechanical basis.

In this section it was shown how the Minkowski position-space appears as reciprocal to the energy-momentum one. In the following section we consider a kinematical meaning of description in Minkowski space-time.

5 A velocity problem and Heisenberg uncertainty principle

The textbook form of the Lorentz (space-time) transformations involves velocity as a purely relative quantity. In currently discussed approach, the notion of velocity refers only to the rest frame of the observer, so that the velocity of observed quantum objects is no longer a relative quantity, what changes essentially the meaning of the well-known transformation formulas.

Before we discuss this issue, we start by considering another velocity aspect, namely, its relationship with particle energy and momentum. As already noticed, one needs to be very careful when uses the velocity notion in the context of momentum space description. An example discussed below reveals the related difficulty.

Derived in the previous section the transformation matrix A_0 (25) was given in terms of parameters

$$\gamma = cosh\xi \quad \text{and} \quad \gamma \cdot \beta = sinh\xi, \tag{45}$$

where, due to (22) and (23), the parameter ξ is directly related to the *active* η. The textbook form of parameters (45) provide the formulas

$$\gamma = \frac{1}{\sqrt{1 - w^2/c^2}} \text{ and } \beta = \frac{w}{c}, \tag{46}$$

which involve some velocity w interpreted as velocity of a particle. Eq. (23) may be then used to express the energy and momentum by means of the velocity w according to

$$E = \frac{mc^2}{\sqrt{1 - w^2/c^2}}, \quad p = \frac{mw}{\sqrt{1 - w^2/c^2}}. \tag{47}$$

However, the parametrization of γ and β in terms of velocity w is not unique. An alternative expressions can be found by introducing a new velocity v related to w by the formula

$$v = \frac{w}{\sqrt{1 - w^2/c^2}}, \tag{48}$$

which yields

$$\gamma = \sqrt{1 + v^2/c^2} \quad \text{and} \quad \gamma \cdot \beta = \frac{v}{c}. \tag{49}$$

As a result, the energy and momentum formulas (47) may be replaced by the new ones

$$E = mc^2\sqrt{1 + v^2/c^2}, \quad p = mv. \tag{50}$$

One sees that the momentum (50) has the classical form, whereas the energy is the regular function of v in the whole range. However, the physical interpretation of expressions (50) is rather troublesome, since the velocities of magnitude grater then c are, in general, not observed. Thus, the arguments supporting the physical meaning of velocity w, but not v, are rather clear.

It is worthwhile to recollect now the two early experimental results that are thought to confirm the interpretative foundations of special relativity, namely the observations of high energy muons [23], and the measurement electron speed in Bertozzi experiment [24].

In the first case it was observed, that as a result of collisions of high energy cosmic radiation protons with nucleus of the upper part atmosphere, the pions are produced. These pions, because of its decay, become a source of abundantly produced muons which state a major part of the secondary cosmic radiation at the sea level. Thus, the distance made by such muon was roughly 20 km. On the other hand, the mean time of muon life established in its rest frame is $\tau \approx 2 \cdot 10^{-6}$ s . For a typical muon energy of 3 GeV, one may put $w \approx c$, which yields the range $c\tau \approx 600$ m only. Thus, to explain this observation on the assumption that velocity of real particle cannot exceed that of light, one needs to assume that the life time of moving muon is increased relative to its life time at rest. The complete explanation of this effect is ascribed to pure geometrical property of the Minkowski space, called the time dilatation effect. According to this, the passage of time in a moving frame is less then in a stationary one.

In the case of Bertozzi experiment, the purpose was to verify the correctness of formula (47). In this experiment the energy of electron ($\lesssim$ 5 MeV) was measured in the calorimetric way and the velocity by use TOF method. Indeed the correctness of formula (47) was confirmed. Nevertheless, both quantities i.e. energy and velocity were established simultaneously and thus, as already noticed, the interpretation of the experimental results obtained this way should have been somehow referred to the Heisenberg uncertainty principle.

Discussed below quantum-mechanical approach to these problems make us possible to avoid a confused (*vital*) time relativity aspect, as well as, to incorporate into the considerations the Heisenberg principle, however, in its generalized form.

Let us assume that a particle seen by the apparatus looks more like an *extended quantum object* than a material point. This might cause that the effective (i.e. measured) time-of-flight interval Δt_w is increased relative to the real one Δt_v because of the interaction between the electron and apparatus. The two (formally equivalent) possible ways of energy and momentum parametrization (47) and (50), allow us to postulate the mutual dependence between both time intervals in the form

$$\Delta l = w\Delta t_w = v\Delta t_v, \tag{51}$$

where Δl is a distance of particle flight, which may be identified either with the total flight distance or just a part of it only (this ambiguity will be explained below). Thus, combining (48) and (51) one finds that the effective and real time intervals of particle flight satisfy

$$\Delta t_w = \frac{\Delta t_v}{\sqrt{1 - w^2/c^2}}. \tag{52}$$

To see that suggested interpretation of (52) is reasonable, firstly, let us assume that the quantum objects we are interested in are really extended, i.e. have some space-time structures. Let us assume next, that we know how such structure looks like, at least for a one particular particle state. We will say that a particle has a *model-shape* if extensions of its ground state (in a way explained below) are characterized by the two position light-cone vectors of the same magnitude

$$\Lambda_0 = \frac{1}{2}\lambda_0, \tag{53}$$

where λ_0 was given in (37). Thus, due to (39) if the object moves, its "light-cone shape" changes from

$$\begin{pmatrix} \Lambda_0 \\ \Lambda_0 \end{pmatrix} \xrightarrow{\text{to}} \begin{pmatrix} \Lambda_1 \\ \Lambda_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\eta}\Lambda_0 \\ \eta\Lambda_0 \end{pmatrix}. \tag{54}$$

However, similarly like in the case of momentum space, we assume now that physically observed quantities are not the light-cone ones but the corresponding them Minkowski "equivalents", which form is given by

$$\begin{pmatrix} \Delta x_0 \\ \Delta x_1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \Lambda_1 \\ \Lambda_2 \end{pmatrix}. \tag{55}$$

The physical meaning of intervals Δx_0 and Δx_1seems to be complement one another. Indeed, let us express these intervals by making use of the formulas

(55), (54) and (46). One easily finds that

$$\Delta x_0 = \frac{1}{\sqrt{1 - w^2/c^2}}\ \lambda_0, \tag{56}$$

$$\Delta x_1 = \frac{w/c}{\sqrt{1 - w^2/c^2}}\ \lambda_0. \tag{57}$$

Note, that $\Delta x_0 \geqslant \Delta x_1$ and in particular, for $w \lessapprox c$, both interval are almost equal, but in the case of $w = 0$, $\Delta x_0 = \lambda_0$ whereas $\Delta x_1 = 0$. This suggests that Δx_1 may be interpreted as the uncertainty of the particle center (of mass) position inside some "quantum region" (which extension will be given latter), and in a time interval resulting from (56). Thus, one may say that the uncertainty Δx_1 concerns a *point-like-particle* position. If this *point-like-particle* moves inside the mentioned quantum region, it is useful to call such motion a movement in a *classical channel.* On the other hand, since Δx_0 must correspond to particle time-like separation, let us assume that it simply corresponds to the time of life of particle quantum state (do not mistake with particle life time), for which the width (or uncertainty) of *classical channel* is Δx_1. Thus, it is useful to call Δx_0 the uncertainty of a *quantum channel.*

To find the explicit dependence between the assumed life times of quantum states for moving particle and particle at rest, let as put

$$\lambda_0 = c\Delta\tau \ \text{ and } \ \Delta x_0 = c\Delta t, \tag{58}$$

so that, the interval $\Delta\tau$ represents the time of life of the ground state of the particle of *model-shape.* Thus, by combining the formulas (58) and (56) one obtains

$$\Delta t = \frac{\Delta\tau}{\sqrt{1 - w^2/c^2}}, \tag{59}$$

which corresponds to the formula (52). Currently, however, the interval Δt is to be interpreted as the life time of the quantum state in which the uncertainty of *point-like particle* position is

$$\Delta x_1 = v\ \Delta\tau = w\Delta t, \tag{60}$$

which, in turn reproduces the formula (51). The special case is, of course, when the times of particle life and its given quantum state are the same. In this case, given approach easily explains mentioned above time dilatation effect.

Discussed approach also provides a new look at particle kinematics. Note, that due to (60) both time intervals $\Delta\tau$ and Δt may serve as *vital time* parameters. Seemingly, i.e. from the classical point of view, only the interval $\Delta\tau$ would be considered as the *vital* one, since it directly concerns the assumed continuous motion of *point-like particle* inside *classical channel*, whereas the interval Δt includes the time at which, naively speaking, *point-like particle* stay at rest, or perhaps it is rather better to say, it oscillates around some stationary point. Nevertheless, although the period Δt encompasses the time at which there is no

propagation along the *classical channel,* it may play a role of *vital time* parameter in a global scale, i.e. if one considers a movement of the whole *extended quantum object* at effective velocity w in a laboratory frame, or in other words in a *quantum channel.* Therefore, neither *classical* nor *quantum* way of movement is smooth but must take a form of "oscillatory" or "jump-like" motion.

This, in turn, leads to a very important conclusion that considered *extended quantum states* may be seen as temporarily localized in a given observer rest frame. This just explains the assumption made above that $\Delta t_w = \Delta t$. On the other hand, one finds that in Minkowski frame identified with given laboratory frame, both time intervals Δt and $\Delta \tau$ find their clear *vital* meanings.

Another aspect of peculiar kind motion outlined above are the momentum fluctuations involved in a mechanism of *classical* and *quantum channel* transmissions. This can be explained within the framework of "updated" Heisenberg uncertainty principle. Indeed, let us consider again the intervals Δx_0 and Δx_1 (56),(57), but now let us write them down in a new but quite equivalent form

$$\Delta x_0 = \frac{h}{\Delta p} + \alpha' \frac{\Delta p}{h}, \tag{61}$$

$$\Delta x_1 = \frac{h}{\Delta q} - \alpha' \frac{\Delta q}{h}, \tag{62}$$

where $\alpha' = (h/mc)^2$ and

$$\Delta p = \frac{h}{\Lambda_2}, \quad \Delta q = \frac{h}{\Lambda_1}. \tag{63}$$

The quantities Δp and Δq are to be interpret as the widths of the momentum uncertainties for the *extended* and *point-like* quantum *objects* just introduced above. In fact, the light-cone relations (63) are already known, since they have been widely discussed in the context of the covariant harmonic oscillator equation [30],[31]. Therefore, there is no problem to establish a correspondence between the introduced light-cone parameters Λ_1 and Λ_2 and the area of maximum density probability distribution for the ground state of (boosted or un-boosted) covariant harmonic oscillator. Furthermore, one easily recognize that Λ_1 and Λ_2 must correspond the size of the *point-like* and *extended quantum object* respectively. Actually, one may go even further and show that the behavior of the "shape" of the whole *extended quantum object* (i.e. with the *point-like object* inside the extended one) is to be describe by O(3)-like group contraction simultaneously to the Euclidean E(2) and cylindrical group [33]. This issue, however needs a separate treatment.

Let us return to the eqs. (61) and (62). Since $\alpha'_m \geqslant \alpha' = (l_P)^2$ and $\Delta x_1 \leqslant \Delta x_0$, one easily finds that

$$\Delta x_0 \geqslant \frac{h}{\Delta p} + \alpha' \frac{\Delta p}{h}, \tag{64}$$

and

$$\Delta x_1 \geqslant \frac{h}{\Delta q} - \alpha' \frac{\Delta q}{h}. \tag{65}$$

Relation (64) is known as generalized Heisenberg uncertainty principle and was proposed by Witten [32] in the context of duality in string theory. The first observation that comes to mind now, is that this duality should correspond a particle-wave duality. The second observation is that the covariant harmonic oscillator may serve as a bridge between the quantum mechanics, spacial relativity and the string theory. Indeed it is hard to resist an impression that the open and closed strings somehow must correspond to unbounded and bounded solutions appearing in the problem of relativistic oscillator. But this issue, of course, goes much beyond the scope of this paper. And finally, what is really now certain, is that special relativity itself preserves the Planck length l_P, however, a particle cannot be seen any longer a material point , but rather as an extended quantum objects endowed with an internal structure.

At the end of this section, let us return to the problem of velocity meaning in the Lorentz transformation formulas. Indeed, the transformation (41) parametrized by means of the formulas (46) yields the known expressions

$$t' = \frac{t - (w/c^2)x}{\sqrt{1 - w^2/c^2}}, \quad x' = \frac{x - wt}{\sqrt{1 - w^2/c^2}}, \quad y' = y, \quad z' = z, \tag{66}$$

interpreted as the transformations of the time and space axes of the two frames moving in the x direction at relative velocity w. The analogical interpretation may be given to the formulas

$$t' = t\sqrt{1 + v^2/c^2} - \frac{v}{c^2}z, \quad z' = z\sqrt{1 + v^2/c^2} - vt, \quad y' = y, \quad z' = z, \tag{67}$$

obtained by means of parametrization (49). However, the *preferred frame description* must refer only to the rest frame of the observer, what, in general, excludes the interpretation assuming that above transformation formulas relate, so called, inertial observers. The both formulas (66) and (67) concern the same *passive* transformation (41), so that it is better to use the *passive* η instead of velocity as the transformation parameter. The exception is the case $\eta \approx 1$ corresponding to small values of w and v. In this situation, the change of *comparison scale* in the *preferred frame description* (given for $\eta = 1$) reduces the formulas (66) and (67) to the Galilean form, and thus provide us the "true" classical limit.

6 *Vital time* as a Lorentz invariant

Within the framework of the above results it is worthwhile to emphasize once more the difference between the *frozen* and *vital* time meanings.

Namely, let us again consider the Lorentz transformations carried out, however, in a light-cone frame, where are called the *squeeze transformations* [34]. In Minkowski frame the two light-cone axes are given by the eqs.

$$x_+ = ct \quad \text{and} \quad x_- = -ct. \tag{68}$$

The physical interpretation of coordinates x_+ and x_- is clear: in the upper light cone part ($t \geqslant 0$) x_+ and x_- are the appropriate distances that a light pulse needs to cover in a time period t.

Now, let us apply *squeeze transformations* to this frame. The axes of the new frame x'_+ and x'_- must satisfy

$$x'_+ = \frac{1}{\eta} x_+ \quad \text{and} \quad x'_- = \eta x_-. \tag{69}$$

But the same, due to (68), may be written in an alternative form

$$t'_+ = \frac{1}{\eta} t \quad \text{and} \quad t'_- = \eta t, \tag{70}$$

which interpretation may confuse us a little bit. Indeed, eqs. (70) cannot be interpret as *vital time* transformations. The vital time meaning can be assign only to the eq. (68), whereas the squeeze transformation (69), in general, destroy this meaning. Nevertheless eq. (70), of course, makes sense but for the *frozen time* meaning. In other words, any *vital time* interval may be always identified with the interval of a light cone axis, which equation in a given Minkowski frame is given by (68). This means, that the *vital time* and so-called proper time is the same.

Having this in mind, let us reexamine the textbook definition of the proper time, which says that the proper time of a system is the time measured by a clock which is stick to the system. One shows, that starting from the interval invariance principle (44), the relation between the proper time τ and the time t, measured in the frame relative to which the "proper clock" moves at the velocity w, is given by

$$\tau = t\sqrt{1 - w^2/c^2}. \tag{71}$$

One then concludes, that the proper time (of a particle) is always less then the relevant time in the stationary frame. Within the framework of presented approach such interpretation, of course, is quite incorrect. This is because the origin of Einstein interpretation is based on quite classical arguments, i.e. on prejudice against the quantum ones [1], which currently constitute the foundations for the present analysis.

Indeed, if one put $ct = \lambda_0$, (where λ_0 is the Compton wavelength) than $h/c\tau$ is the corresponding particle energy, i.e. the energy of a particle, which in a laboratory frame moves at a velocity w. On the other hand, if one looks at the eq. (71) as the relation between the *vital time* intervals, then its interpretation needs to be supported by quantum-mechanical analysis given in the preceding section. One should notice, however, that now $\tau \to \Delta t$ and $t \to \Delta\tau$ (cf. eq. (59)). Therefore, the both time intervals indeed have the *vital* meaning, however, this concerns only the preferred Minkowski-observer frame. Thus, in general, one finds that the time that undergo relativistic transformation rules is the *frozen* one, whereas the *vital* one is the Lorentz invariant.

7 Concluding remarks

Excepting the gravity issue, the basic tools of physical description are: the Newton theory, quantum mechanics and special relativity. Although these three theories have been developed as quite independent ones, currently they provide the basis of the quantum field theory. Of course, it is commonly thought that the known Dirac equation is the element that joins the two separate realms of relativity and quantum mechanics. Nevertheless, the interpretation of the Lorentz symmetry presents simply a generalization of the interpretation of the Galilean transformation in the Newton theory, what finds its reflection in the first Einstein relativity postulate. The main purpose of this paper was to suggest that the origin of the Lorentz symmetry is rather quantum then classical.

This outlook results from given elementary analysis embracing the idea of *preferred frame description* (and observation), the concept of *relative scaling,* and the *light-cone skeleton* able to encompass basic informations about external and internal particle features. In our analysis these external features were particle energy and momentum, whereas the internal ones were particle space-time extensions.

It has been shown that the *relative scaling* combined with the principles of quantum mechanics yield clear interpretation of Lorentz transformations, devoid of a tinge of guessing. Let us note, that the interpretation of the Lorentz symmetry as the consequence of the relativity principle had come as a result of the replacement of the Galilean transformation by the Lorentz one, in a situation when mathematical structure of the Maxwell equations had made impossible to apply the Galilean transformation. However, there had been significant differences in foundations of both (i.e. Newton and Maxwell) theories, which had caused that such generalization might be not justifiable. For example, in the Newton theory we deal with the concept of material point, which does not have a simple counterpart in the wave-like description. Although the Maxwell equations are seen as the classical ones (and the Plank constant cancels out from the photon dispersion relation), the simple physical observations, such as the photoelectric effect, or the blackbody radiation, revel the quantum structure of the electromagnetic field. Thus, one may say that the electromagnetic field is a system in which the classical and quantum features are somehow "smeared out". Nevertheless, the Lorentz symmetry so far is always thought as a manifestation of the classical (space-time) property. The analysis given in the paper suggests, that just the quantum features of physical systems decide that Lorentz or Poincaré symmetry comes into play.

Briefly, there are two separate ways of approaching the matter of relativistic quantum physics. The first, well-known, assumes relativity of frame description, relativity of time and existence of material point-like objects. The other, currently proposed, assumes preferred frame description, absolute time and extended quantum objects with internal structures. One of the consequences of the latter approach is that special relativity, in a natural way, becomes an integral part of quantum mechanics.

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